

Key Generation

1. Select two prime numbers: 23, 31
2. $N = 23 * 31 = 713$
3. $\Phi(N) = (23-1) * (31-1) = 660$
4. e has to be in the range of {1...659} and e is coprime with 713 and 660

$$713 = 23 * 31$$

660 can be divided by 2,3,5...

So, 713 and 660 both have no common factors with the value 91

$$E = 91$$

5. $d = (k * \Phi(N) - 1) / e$
[k can be any integer value for which the answer of d is also an integer]
 $= (4 * 660 - 1) / 91 = 29$

$$23 * 31 = 713$$

$$\begin{aligned}\Phi(713) &= \Phi(23) * \Phi(31) \\ &= (23 - 1) * (31 - 1) \\ &= 660\end{aligned}$$

Coprime: does not share any common factors.

14 is coprime with 15 and 23

Phi function refers to how many integers are less than or equal to N that do not share any common factors with N.

$$7 = 7 * 1$$

7 can be divided by 7

How many integers are less than or equal to 7? => 7

So, $\Phi(7) = 6$

For every prime number, the phi function will be (Prime number - 1)

Public Key {3,3127}

Private Key {2011,3127}

$$m^e \pmod{N} = C$$

We have to encrypt the message "Hi"

$$Hi = 89$$

$$89^3 \pmod{3127} = 1394$$

Where $m = 89$, $e = 3$, $N = 3127$

$$c^d \pmod{N} = m$$

$$1394^{2011} \pmod{3127} = 89$$

So, using the value of the private key, d; Alice can easily decrypt the ciphertext